Creeping motion of a particle along the axis of axisymmetric containers

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Hydrodynamic interactions between particles and walls are relevant for the open problem of specifying boundary conditions for suspensions flows. The Reynolds number around a small particle close to a wall is usually low and creeping flow equations apply. In this presentation, I will focus on hydrodynamic interactions between a settling particle and walls when the normalized distance with walls is larger or lower than unity.

In the far-wall hydrodynamic interactions between a settling particle and walls, several types of containers are considered here: circular cylinders closed by planes at both ends, cones closed by a base plane, ... The axis of the container is set in a vertical position. The cone tip is pointing down. The particle is solid and spheroidal (sphere, sphere slightly deformed, sphero-cylinder, ...). It is settling along the container axis so that the fluid motion is axisymmetric. The Stokes flow problem is solved numerically, using a technique pioneered by Bourot (1969) and used thereafter in various problems by Coutanceau and coworkers. The solution of Stokes equation for the fluid velocity is written as a series in spherical coordinates around the sphere and the boundary condition on the sphere is applied exactly. The boundary condition on the walls of the container then is applied in the sense of least squares: the quantity to minimise is written as an integral on the boundary of the squared difference between the approximated velocity and the value to be enforced. The minimization provides the coefficients in the series. Calculated streamlines patterns for a small sphere in a cylinder are in agreement with results by Blake (1979) for a Stokeslet in a cylinder but differed from the ones obtained by Sano (1987). Various sets of eddies appear in cylinders and cones, depending upon the geometry and the sphere position. Results are in agreement with earlier works about eddies in close containers and corners when in Stokes flow (Moffatt 1964, O'Neill 1964, O'Neill 1983). With a standard computer accuracy, the present numerical technique applies when the gap between the sphere and the nearby wall is larger than about one radius.

For a sphere in the vicinity of any of the plane walls, we match our results with the analytical solution of Brenner (1961) and Maude (1961). Our results for the drag force supplemented with the analytical solution of Brenner (1961) and Maude (1961) near the plane walls (at distances typically less than a diameter, depending on the sphere size) are in excellent agreement with the experimental data for the cylindrical and the conical containers. Experiments show that the motion towards the apex of a cone is much slower than that towards a plane. This is because of the hindered backflow. For a dimensionless gap (normalized with the sphere radius) much smaller than unity, the drag force in a cone varies like the normalized distance to the power (-5/2), in agreement with the lubrication result of Masmoudi et al (1998), whereas it varies like the normalized distance to the power (-1) close to a plane (Lecoq et al (2007)). Wakiya (1976) presented the general features of a three-dimensional axisymmetric flow in a space with a conical boundary. His solution near the apex reveals features similar to Moffatt's flow (see Moffatt (1964)). In the same way as in the two-dimensional case, Wakiya showed the existence of an infinite sequence of eddies near the apex for certain values of the semi-angle of the cone, which are in general less than about 80.9^o. The Stokes assumption is valid sufficiently near the apex of the cone and the behaviour of the fluid there is to some extent independent to the nature of the far field. In the general solutions presented in details here are considered separately the presence or absence of a dead water area in the apex of the cone. The main differences between both solution are discussed. Then, the theoretical solutions are compared with the experimental measurements for cones with various semi-angle at the apex (90^o (plane wall), 89^o, 88^o, 85^o, 80^o, 65^o, 45^o). For all the cases, the theoretical results for the normalized velocity were found to be in very good agreement with experiments (Lecoq and Feuillebois (2007)).

From the solution of the creeping-flow equations, the drag force on a sphere becomes infinite when the gap between the sphere and a smooth wall vanishes at constant velocity, so that if the sphere is displaced towards the wall with a constant applied force, contact theoretically may not occur. Physically, the drag is finite for various reasons, one being the particle and wall roughness. Then, for vanishing gap, even though some layers of fluid molecules may be left between the particle and wall roughness peaks, conventionally it may be said that contact occurs. This physical importance of roughness provide thus a strong motivation for studying the hydrodynamic of suspension with rough surface. In this last part of the presentation, we are considering the example of a smooth sphere moving towards a rough wall. Smart & Leighton (1989) measured the hydrodynamic effect of the surface roughness of a sphere moving perpendicularly to a smooth wall. Some of their spheres were made rough by gluing very small spheres on their surfaces. Here we make the reverse, that is we prepare walls with a definite roughness, and the sphere roughness is small in comparison. The roughness considered here consists of parallel periodic wedges, the wavelength of which is small compared with the sphere radius. This problem is considered both experimentally and theoretically (Lecoq et al (2004)).

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