Motion of an ellipsoidal particle in a shear flow along a porous slab

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Abstract

This work examines the motion of a solid and non-necessarily-spherical particle with surface S near the upper boundary Σ_1 of a plane porous slab. The flow is governed by the Stokes equations in the fluid domains and the Darcy's equations in the porous medium with permeability $K \geq 0$. These equations are coupled across the porous slab upper and lower plane boundaries Σ_1 and Σ_0 by a slip boundary condition (BJ) proposed by Beavers and Joseph [1].
A modified boundary-integral approach, avoiding integrals on the surface Σ_1, is used to obtain the fluid velocity in the liquid domain and the net force and torque exerted by the flow on the particle surface [2, 3].
Numerical results are presented and discussed for an ellipsoidal solid particle that is translating and rotating when settling under the action of a prescribed uniform gravity field $\mathbf{g}$ or freely suspended in a linear or quadratic ambient shear flow with velocity $\mathbf{u}_\infty$ along the wall Σ_1. The ellipsoidal particle has center O', semi-axes $a_1 = 1.2a$, $a_2 = 1/a$ $a_3 = a$ and its orientation with respect to Σ_1 is given by the angle $\alpha = \pi/4$. For conciseness, we restrict our attention to rigid-body motions $(\mathbf{U}, \mathbf{\Omega})$ such that $\mathbf{U}.\mathbf{e}_2 = 0$ and $\mathbf{\Omega} \wedge \mathbf{e}_2 = \mathbf{0}$ (where $\mathbf{e}_2$ is a unit vector tangent to Σ_1 and parallel to the ambient vorticity). In such circumstances and adopting the scaling employed for a sphere having the same volume (i.e with radius a) [4, 5], we normalize the drag force, torque, translational and rotational velocities as follows:

$$\mathbf{F}_h = -6\pi\mu a\{\{f_{11}(\mathbf{U}.\mathbf{e}_1) + f_{13}(\mathbf{U}.\mathbf{e}_3)\}\mathbf{e}_1 + af_{12}(\mathbf{\Omega}.\mathbf{e}_2)\mathbf{e}_1 + \{f_{33}(\mathbf{U}.\mathbf{e}_3) + f_{31}(\mathbf{U}.\mathbf{e}_1)\}\mathbf{e}_3 + af_{32}(\mathbf{\Omega}.\mathbf{e}_2)\mathbf{e}_3\}, \quad (1)$$

$$\mathbf{\Gamma}_h = -8\pi\mu a^2\{\{c_{21}(\mathbf{U}.\mathbf{e}_1) + c_{23}(\mathbf{U}.\mathbf{e}_3)\}\mathbf{e}_2 + ac_{22}(\mathbf{\Omega}.\mathbf{e}_2)\mathbf{e}_2\}, \quad (2)$$

$$\mathbf{U} = \frac{2}{9}\frac{a^2}{\mu}(\rho_S - \rho)\{\{u^s_{11}\mathbf{e}_1 + u^s_{31}\mathbf{e}_3\}(\mathbf{g}.\mathbf{e}_1) + \{u^s_{13}\mathbf{e}_1 + u^s_{33}\mathbf{e}_3\}(\mathbf{g}.\mathbf{e}_3)\}, \text{ if } \mathbf{u}_\infty = 0, \quad (3)$$

$$\mathbf{\Omega} = \frac{2}{9}\frac{a}{\mu}(\rho_s - \rho)\{\omega^s_{21}(\mathbf{g}.\mathbf{e}_1) + \omega^s_{23}(\mathbf{g}.\mathbf{e}_3)\}\mathbf{e}_2, \text{ if } \mathbf{u}_\infty = 0, \quad (4)$$

$$\mathbf{U} = \gamma_1(l + \frac{\sqrt{K}}{\sigma})\{u^l_1\mathbf{e}_1 + u^l_3\mathbf{e}_3\} + \gamma_2(l^2 + \frac{a^2}{3} - 2K)\{u^q_1\mathbf{e}_1 + u^q_3\mathbf{e}_3\}, \text{ if } \mathbf{g} = \mathbf{0}, \quad (5)$$

$$\mathbf{\Omega} = (\frac{\gamma_1}{2}w^l_2 + \gamma_2 l w^q_2)\mathbf{e}_2, \text{ if } \mathbf{g} = \mathbf{0}. \quad (6)$$

By comparison with a sphere, nine additional dimensionless quantities appear: f_{31}, f_{13}, f_{32}, c_{23}, u^s_{31}, u^s_{13}, w^s_{23}, u^l_3 and u^q_3. These new quantities (which vanish for symmetry reasons for a sphere) obey the relationships $f_{13} = f_{31}$ and $4c_{23} = 3f_{32}$ for arbitrarily-shaped particles when $K = 0$ (K denoting the permeability of the porous slab) or for a distant porous slab. In general for the (BJ) slip condition one has $f_{13} \neq f_{31}$, $4c_{23} \neq 3f_{32}$ and $4c_{21} \neq 3f_{12}$. The talk will present the sensitivity of the normalized quantities to K and to the ellipsoid location.

1. References

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